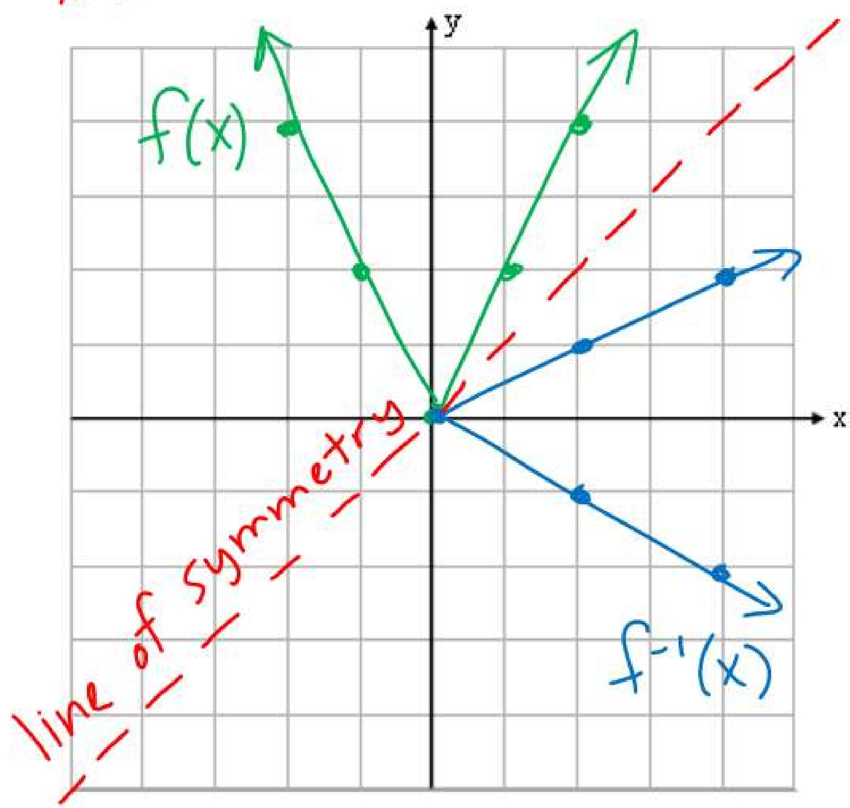
2.8 NOTES: Inverse Functions

example: graph $f(x) = |2x|$ *and* its inverse

inverse notation

x	f(x)
-2	4
-1	2
0	0
1	2
2	4

x	$f^{-1}(x)$
4	-2
2	-1
0	0
2	1
4	2



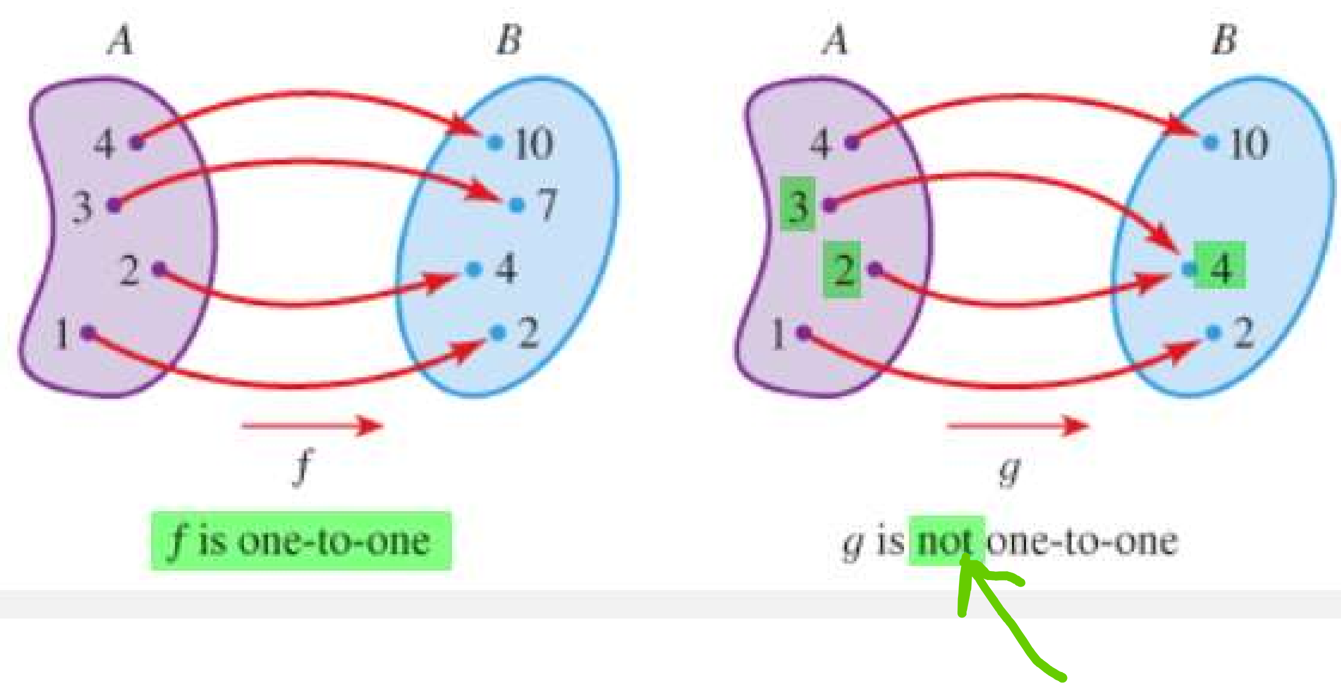
domain values become the range values + vice versa

Note: f(x) and $f^{-1}(x)$ are symmetrical to each other with respect to the line y = x [or f(x) = x]

One-to-one function:

Each domain value has a unique range value.

Figure 1



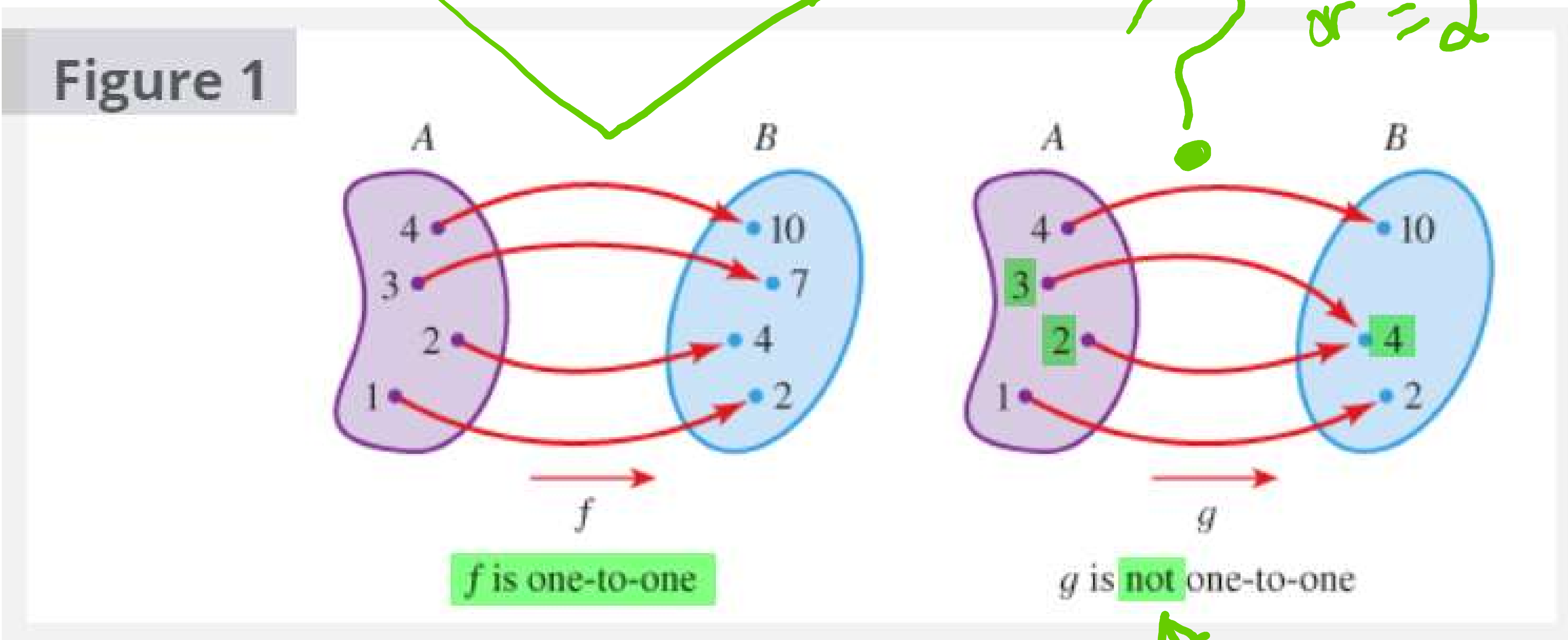
One-to-one function:

Each domain value has a unique range value.

example : $f(3) = 7 \qquad f^{-1}(7) = 3$

$f(2) = 4 \qquad f^{-1}(4) = 2$

$f^{-1}(4) = 3$ or $= 2$



To solve for the inverse algebraically:

"swap" x and y values in the given equation, then solve the equation for y (rewrite in y-form.)

example: $f(x) = \frac{4}{13+x}$

rewrite: $y = \frac{4}{13+x}$

swap domain and range: $x = \frac{4}{13+y}$

now solve for y

swap domain and range: $x = \frac{4}{13+y}$

now solve for y

$$(13+y)x = \frac{4}{(13+y)}(13+y)$$

$$(13+y)x = 4$$

$$13+y = \frac{4}{x}$$

$$y = \frac{4}{x} - 13$$

$$f^{-1}(x) = \frac{4}{x} - 13$$

domain: $x \neq 0$

CHECK EVEN ANSWERS:

26. a) $f(5) = 18 \rightarrow f^{-1}(18) = \boxed{5}$

    b) $f(4) = 2 \rightarrow f^{-1}(2) = \boxed{4}$

28. find g^{-1} by solving $x^2 + 4x = 5$ and using zero product property

so if $g(1) = 5 \rightarrow f^{-1}(5) = \boxed{1}$

CHECK EVEN ANSWERS:

30. a) $g^{-1}(2) \rightarrow \text{think}: \; g(\quad) = 2$

$g(4) = 2$ from the graph

therefore $g^{-1}(2) = \boxed{4}$

b) $g^{-1}(5) \rightarrow$ $g(7) = 5$ from the graph

therefore $g^{-1}(5) = \boxed{7}$

CHECK EVEN ANSWERS:

30 c) $g^{-1}(6) \rightarrow$ $g(8) = 6$ from the graph

therefore $g^{-1}(6) = \boxed{8}$

32) $f^{-1}(0) \rightarrow$ $f(5) = 0$ from the graph

therefore $f^{-1}(0) = \boxed{5}$